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The University of Massachusetts
Department of Mathematics and Statistics
Amherst, Massachusetts

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A Characterization of a Polya-Eggerberger and Other Discrete Distributions by Record Values

Ramesh M. Korwar

University of Massachusetts

Abstract

Let X_1, X_2, ..., be a sequence of independent and identically distributed discrete random variables. Define the sequence $\{L(n)\}$ by $L(1)=1$, $L(n)=\min\{j \mid j > L(n-1), X_j > X_{L(n-1)}\}$, $n=2$, ... Let $R_n=X_{L(n)}$. Then $\{R_n\}$ is the sequence of record values. By convention $R_1=X_1$. Here a characterization of a Polya-Eggenberger and other discrete distributions including the geometric, is made by the linearity of regression of R_2-R_1 on R_1.

A CHARACTERIZATION OF A
POLYA-EGGENBERGER AND OTHER DISCRETE
DISTRIBUTIONS BY RECORD VALUES [1]

by

R M KORWAR
Indian Statistical Institute
and
University of Massachusetts

SUMMARY. Let $X_1, X_2, \ldots,$ be a sequence of independent and identically distributed discrete random variables. Define the sequence $\{N(n)\}$ by $N(1)=1$, $N(n)=\min\{j \mid j>N(n-1), X_j>X_N(n-1)\}$, $n=2,3,\ldots$. Let $R_n=X_{N(n)}$. Then $\{R_n\}$ is the sequence of record values. By convention $R_1=X_1$. Here a characterization of a Polya-Eggenberger and other discrete distributions including the geometric, is made by the linearity of regression of R_2-R_1 on R_1.

Consider a sequence $X, X_1, X_2, \ldots$ of independent and identically distributed (i.i.d.) discrete random variables (r.v.) with

$$P(X=j) = p_j \ , j=0,\ldots,m \qquad (1)$$

Here m is either a positive integer or ∞. Define $N(1)=1$, $N(n)=\min\{j \mid j>N(n-1),\ X_j>X_{N(n-1)}\}$, $n=2,3,\ldots,$ $R_1=X_1$ and $R_n=X_{N(n)}$, $n=2,3,\ldots$. Then $\{R_n\}$ is the sequence of record values and $\{N(n)\}$ the sequence of times at which record values occur.

In this note we characterize the gemetric, a Polya-Eggenberger and a generalized hypergeometric distribution by linearity of regression of R_2-R_1 on R_1. Thus the

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characterization given here extends Theorem 2 of Srivastava (1979) characterizing the geometric by the constancy of regression of R_2-R_1 on R_1.

Consider for X one of the following distributions.

$$p_j=(1-p_o)\, c\{1-c\}^{j-1},\ j=1,2,\dots, \qquad (2)$$

$o < c < 1.$

$$p_j = (1-p_o)\binom{a}{j-1}\binom{b}{n-j+1}\Big/\binom{a+b}{n},\ j=1,\dots,n+1, \qquad (3)$$

$a < o;\ n > o;$ n integral; $n-b-1 > o;\ b < o,\ b \neq -1.$

and

$$p_j = (1-p_o)\binom{a}{j-1}\binom{b}{n-j+1}\Big/\binom{a+b}{n},\ j=1,2,\dots,$$

$a < o;\ n < o;\ n-b-1 < o;\ a+b+1 > o. \qquad (3a)$

Distribution (2) is a geometric distribution, (3) and (3a) are versions of generalized hypergeometric distribution; Type 2A and Type IV, as described by Kemp and Kemp (1956). For a=-1, (3) can be recast as

$$p_j = (1-p_o)\binom{n}{j-1}\frac{A(A+s),\dots,(A+\overline{j-2}s)\ B(B+s),\dots,(B+\overline{n-(j-1)-1}s)}{(A+B)\ (A+B+s)\ ,\dots,(A+B+\overline{n-1}\ s)}$$

$$j=1,\dots,n+1 \qquad (3b)$$

with A=1 , B=-b, s=1. An example of a Polya-Eggenberger distribution is (3b).

We are now ready to prove the

Theorem: Suppose X takes on only the values o,...,m with positive probabilities and assume that X has a finite expectation. Then

$$E(R_2-R_1 \mid R_1=i) = \alpha+i\beta \quad \text{a.s.}$$
$$\alpha,\beta \text{ Constants} \qquad (4)$$
$$i=o,\ldots,m-1$$

if, and only if, X has either the geometric (2), or a Polya-Eggenberger (3b) with A=1 and s=1, or a generalized hypergeometric distribution (3a) with a=-1. Furthermore, $\beta=o$ iff X is geometric (2), $\beta < o$, $\beta \neq 1$ iff X is (3b) with A=1 , s=1 and $\beta > o$ iff X is (3a) with a=-1.

Proof: First consider the ''only if'' part. Suppose (4) holds. Now it is shown in Srivastava (1979) that

$$E(R_2-R_1 / R_1=i) = \sum_{j=1}^{m-i} p_{i+j} \Big/ \sum_{j=i+1}^{m} p_j \ , \quad i=o,\ldots,m-1$$

Thus it follows that

$$\sum_{j=1}^{m-i} jp_{i+j} = (\alpha+i\beta) \sum_{j=i+1}^{m} p_j \qquad (5)$$

and

$$\sum_{j=1}^{m-i-1} jp_{i+j+1} = (\alpha+\overline{i+1}\ \beta) \sum_{j=i+2}^{m} p_j \qquad (6)$$

Subtracting (6) from (5) we have

$$(1+\beta) \sum_{j=i+1}^{m} p_j = (\alpha+ \overline{i+1}\ \beta)\ p_{i+1} \qquad (7)$$

Replacing i by (i+1) in (7) and substracting the result from (7), we finally obtain

$$p_{i+2} / p_{i+1} = (\alpha+\beta i-1) / \{ \alpha+\beta(i+2) \} \ , \quad i=0,\ 1,\ldots.m-2.$$

This last result and (7) yield

$$p_j=p_1\,\frac{(\alpha-1)\;.\;.\;.\;(\alpha+\overline{j-2}\beta-1)}{(\alpha+2\beta).\;.\;.\;(\alpha+j\beta)}$$

$$j=1,\ldots,m. \tag{8}$$

and

$$p_1=(1+\beta)\;(1-p_0)\;/\;(\alpha+\beta). \tag{9}$$

From (4) it follows that

$$\alpha \geq 1 \tag{10}$$

We consider three cases (i) $\beta=0$, (ii) $\beta<0$ and (iii) $\beta>0$. The case $\beta=0$ is already considered by Srivastava (1979). His result is included in (8) and specializing for $\beta=0$, (8) and (9) give (2) with $c=1/\alpha$.

Next consider the cases $\beta\neq 0$. Now, from (4), it follows that

$$\beta < 0 \quad \text{if, and only if, X is bounded} \tag{11}$$

$$\text{If } \beta < 0 \quad \text{then } \alpha+\beta(m-1)=1 \tag{12}$$

We obtain (12) from (4) by specilizing $i=m-1$. From (8) and (9) it is clear that $\beta=-1$ leads to the degeneracy of X at 0, a contradition of the assumption of m being a positive integer. For the cases $\beta\neq 0$ (8) can be recast as

$$p_j=p_1\left\{A^{[j-1]}\;B^{[j-1]}\;/\;C^{[j-1]}\;(j-1)!\right\},\; j=1,\ldots m \tag{13}$$

where $x^{[j]}$ is the ascening factorial,

$$x^{[j]}=x\,(x+1)\ldots(x+j-1),\; j=1,\,2,\ldots,\; x^{[0]}=1;$$

p_1 is given by (9) and

$$A=(\alpha-1)/\beta \qquad B=1\;,\;\text{and } C=\alpha/\beta+2. \tag{14}$$

The product in the curled brackets in (13) is the coefficient of z^{j-1} in the hypergeometric function

$$F(A;B;C;z) = \sum_{j=0}^{\infty} \left\{ A^{[j]} B^{[j]} / C^{[j]} j! \right\} z^j.$$

Thus (13) is the generalized hypergeometric distribution (3) (or 3(a)) with

$$a=-1, \quad b=1+1/\beta, \quad n=-(\alpha-1)/\beta \tag{15}$$

First suppose $\beta<0$. Then, from (12) and (15) we have that $n=m-1$, and if $m\geq 2$, from (9), that $b<0$. Obviously, $n-b-1>0$ for $m\geq 2$, Thus, for the case $\beta<0$, $\beta\neq 1$ and $m\geq 2$ we have a generalized hypergeometric distribution (3) with parameters $a=-1$, $b=1+1/\beta$ and $n=m-1$. This distribution is also a Polya-Eggenberge distribution (3b) with parameter $A=1$, $B=-1/\beta$, $n=m-1$ and $s=1$. However, (4) will not lead to any particular distribution if $m=1$.

Finally assume $\beta>0$. From (15), we have that X has the generalized hypergemetric distribution (3a) with the parameters given by (15).

Now for the ''if'' part. Let X have (3) or (3a) with $a=-1$. Then we have

$$p_{j+1}/p_j = (j-1-n) / (b-n+j), \quad j=1,2,\ldots, m-1.$$

from which we get

$$\sum_{j=k+1}^{m} jp_j \Big/ \sum_{m=k+1}^{m} p_j = (b-n-1+bk) / (b-1), \quad k=0,\ldots,m-1$$

which in turn yields (4) with $\alpha=(b-n-1)/(b-1)$ and $\beta=b/(b-1)$. Hence the ''if'' part is proved.

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